

[45] Average (or Mean)

There are technically at least three different types of averages – mean, median, and mode. They will all be covered in a later lesson. Here, only the mean is covered. In general conversations, when people refer to “the average” they are usually referring to the “mean”.

To average is to even things out so that you find out how much there would be if everyone got the same amount.

Teaching Ideas

To explain average, you can say that “*averages make things fair.*” You can teach the concept of average even before you teach division, and you can even bring younger siblings into the demonstration. Children who haven’t been taught division can still understand average and figure averages using manipulatives. (They do this all the time – they just don’t know the terms.)

Take a handful of something small that can be easily divided. Ideas include small edible items, such as nuts, crackers, pretzels, small cookies or candies; small toys, such as Legos® or jacks; or pennies. Make sure that you start with an amount of items that can be evenly divided by the number of participants. *At the end, each person should be able to have the same amount.*

NOTE: Be sure to use items that are safe for the ages of the children involved.

Give each person a *different* amount and let each person count them (if edible, don’t eat yet) and compare — have the items been distributed *fairly*? Does everybody have the same amount? We can average to make it fair. Here is how.

- a. Put everything in the middle and add them together.
- b. Now divide the items up evenly, putting one piece at a time in front of each person.

That is really all that averaging means — add the items all together, then divide them evenly. The divisor is determined by counting how many participants there are in the demonstration.

$$\text{Average} = \frac{\text{Total amount}}{\text{Number of participants}}$$

Here are the steps to use when practicing averaging.

1. Practice averaging using manipulatives.
2. Average with manipulatives, but also write down what you are doing.

Example: 3 people, one gets 10 items, one 6 items, and one 14 items.

Write down on the board:

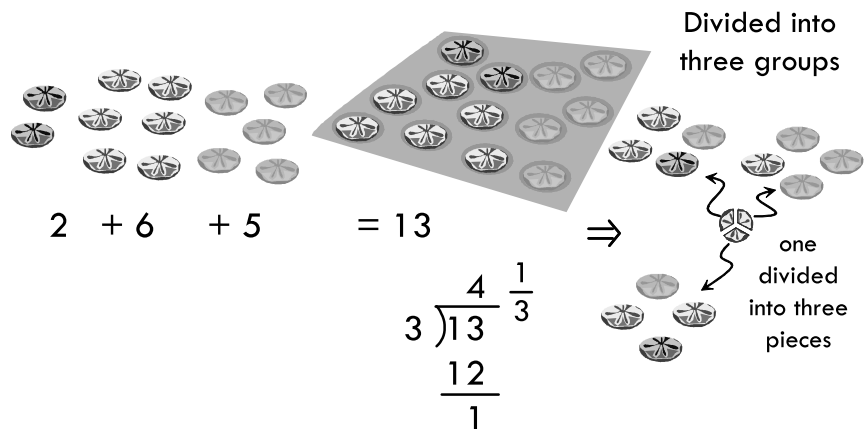
$$\begin{array}{r} 10 \\ + 6 \\ + 14 \\ \hline 30 \end{array} \qquad \begin{array}{r} 10 \\ 3 \overline{)30} \end{array}$$

NOTE: If your averaging lessons include beginning math students, you may want to keep them working with manipulatives.

3. After you have done this several times, then average numbers without using manipulatives.
4. The last step is to teach your child what to do with remainders. (This is also a good tie-in to fractions.)

Handling Remainders

Average a group of numbers, such as 2, 6, and 5. (You can do this with manipulative also — brownies or cookies work well, dividing them among three people.)



There is a remainder of one. In order to divide that remaining one evenly among three, it must be divided into three pieces, and each person gets one piece, or $\frac{1}{3}$. If you were to repeat the averaging process with 3, 6, and 5, there would be a remainder of two. If those two were cut into three pieces each, there would be two pieces for each person.

$$3 \overline{)14} \begin{array}{r} 4 \\ \underline{12} \\ 2 \end{array}$$

To handle remainders when averaging, put the remainder as the numerator and the divisor as the denominator.

[46] Mean, Median, Mode and Range

*Mean is what is usually referred to by **average**. It is equal to the sum of the numbers divided by the quantity of the numbers.*

*Median is the number in the **middle** when the values are arranged in order. If your children know the median is the center strip in a road, then the term “median” will make more sense.*

*Mode is the **most common** value among the group of numbers. The word “mode” can describe what people are most often doing (or wearing). In both cases, the term “mode” refers to what is most common. For our purposes, we will only use modes when at least one number occurs more than once.*

*Range is the **difference** between the lowest value and the highest value.*

These terms are different ways of describing a set of numbers. Mean, Median, and Mode are different types of averages, although they give three different kinds of information. We will first look at finding each of these three types of average for the following set of numbers: { 1, 5, 7, 7, 10 }.

Mean

Mean (also called arithmetic mean) is the mathematical term for *average*. This term is covered in [45] Average (or Mean), page Op:82. The *mean* of these numbers is equal to the sum of the numbers divided by the quantity of numbers.

$$1 + 5 + 7 + 7 + 10 = 30 \quad \text{and} \quad 30 \div 5 = 6$$

Median

When the numbers in a set are written in order, the median is the number in the middle. It is the value that separates the lower half of the numbers from the upper half. For an odd set of numbers, finding the median is easy — it is the number in the very middle of the group (when the numbers are arranged in order). For example, in the set { 1, 5, 7, 7, 10 }, the median (or middle number) is 7.

When dealing with an even set of numbers, no one value is in the very middle, so finding the median takes an extra step (to find the average of the two middle numbers).

For example, to find the median of { 24, 7, 4, 20, 1, 3 }, first arrange the numbers in order: { 1, 3, 4, 7, 20, 24 }. Find the two numbers in the middle (4 and 7), add them, and divide by two to find the mid-way value.

$$\text{median} = \frac{4+7}{2} = \frac{11}{2} = 5.5 \text{ (or } 5\frac{1}{2} \text{)}$$

Mode

In math, *mode* is the number that occurs most often in a set of numbers. In the previous example, the number 7 occurs most often (there are two of them, and only one of each of the other numbers); therefore, the *mode* is 7.

Sometimes a group of numbers will have more than one mode. In this set of numbers {3, 5, 6, 6, 6, 7, 7, 8, 8, 8, 9} there are two modes. In this set, the number 6 occurs three times, but the number 8 also occurs three times. Therefore, both 6 and 8 are *modes*.

Range

Range is used to describe the distance between the lowest number and the highest number — it is the difference between one end and the other. To find the range, just subtract the lowest number from the highest number. In our first example set of numbers, the lowest is 1 and the highest is 10, so the range is 9 (= 10 – 1). In the third example set, the range is 6 (= 9 – 3).

[47] Dividing: Two or more Digits in Divisor

This skill can be taught right after rounding/ estimating (see [107] Rounding and Estimating). Otherwise, make sure the child has a good understanding of rounding/ estimating before teaching this skill.

$$\begin{array}{r} \text{quotient} \\ \text{divisor} \overline{) \text{dividend}} \end{array}$$

This lesson teaches long division when the divisor is 10 or greater.

Teaching Ideas

When doing long division, use graph paper, or binder paper turned sideways, to keep all numbers lined up. Long division is just a series of little division problems.

1. Find the first little problem.

Figure out how many digits will be used in the dividend of the first problem. In other words, determine how many digits are needed to make groups that are as big or bigger than the divisor.

$$\begin{array}{r} 0 \\ 37 \overline{) 948} \end{array} \quad \begin{array}{r} 0 \\ 37 \overline{) 948} \end{array}$$

Can you make groups of 37 with 9? (No.) Therefore, put 0 over the 9.

Can you make groups of 37 with 94? (Yes.) Start the division problem with 94.

Now use the same steps as were used with single-digit divisor division — *divide, multiply, subtract, bring down*.

2. **Divide**

To determine the answer to the “little division problem,” here are two approaches to try.

- a. Estimate. Round the divisor and see how many times the rounded number goes into the dividend. Round 37 to get 40; then use 4 into 9 or 40 into 90.

$$\begin{array}{r} 0 \\ 37 \overline{) 948} \end{array}$$

- b. Guess and check: take a guess at how many times the divisor goes into the dividend. **KEEP A RECORD.** Multiply your guess by the divisor and see if it fits into the dividend. If it is too big, then try a smaller number. If it fits, then subtract and make sure that the difference is less than the divisor. If it's more than the divisor, then try a bigger number.

NOTE: Keep a record: On scratch paper, keep a list of all the attempts you make, like this.

$$\begin{array}{r} 37 \\ \times 3 \\ \hline 111 \end{array} \quad \begin{array}{r} 37 \\ \times 2 \\ \hline 74 \end{array}$$

(You can use this list for the entire problem).

3. **Multiply** the quotient by the divisor and put the product under the dividend.