[83] Area of a Triangle

The area of a triangle is calculated with the formula:

A=½bh

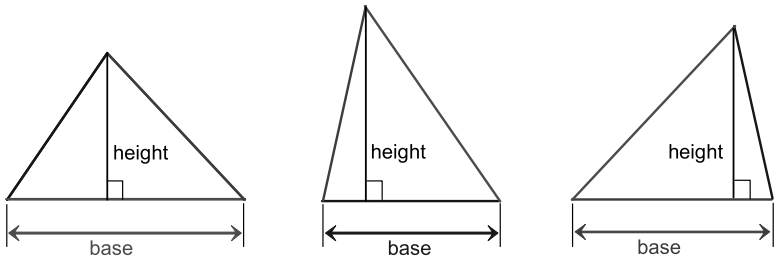
where A is area,
b is the base (bottom dimension), and
h is the height of the triangle.

The base and the height change depending on how the triangle is oriented, but the area formula will still work.

The area of a triangle is a simple formula to memorize. However, it is helpful if your child understands *why* the formula works. Therefore, this lesson teaches the formula and also gives activities to help *understand* the formula.

Vocabulary: Base and Height

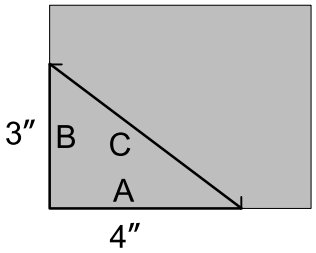
The two basic terms to learn regarding triangles are *base* and *height*. The bottom dimension of the triangle is called its *base.* The base isn't a permanent feature of the triangle; the side that we call the base depends on how the triangle is oriented. You can turn a triangle around so that any of its three sides is on the bottom, so any of its sides can be the *base*.



The *height* of the triangle is the length of a line that is perpendicular to the base and extends from the base to the top of the triangle. Like the base, the *height* of the triangle depends on how the triangle is oriented.

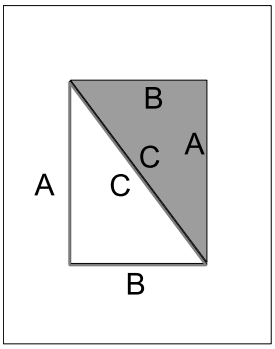
Teaching ideas

To help your child understand this concept, have him or her make a right triangle out of colored tagboard or construction paper. Make a mark four inches from the corner along the bottom of the paper and label that side A. Make another three inches up from the same corner and label that side B. Connect the two marks with a line and label that line C. Then carefully cut out the triangle.



Have the child turn the triangle different ways. Point out that each of the three sides can be the base, and for each base, the height is different.

Now turn the triangle so that side A is the height and side B is the base. Notice that it this makes a right triangle, which makes the height equal to side A. Put the triangle in this position on a piece of typing paper and trace the triangle. Then turn the triangle around so that side B is at the top of the paper and side C is along the side C that you traced before. Now trace the triangle again and look at the result; the two triangles together form a rectangle.



The area of a rectangle equals the product of its two sides, $A \cdot B$. Because the inside of the rectangle contains exactly two identical triangles, the area of one of the triangles must equal one-half of the area of the rectangle, or ½ of side B times side A, or ½ × *base* × *height*.

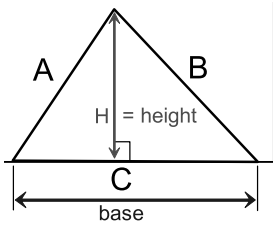
$$A = \frac{1}{2} \cdot BH$$

From the commutative property of multiplication, it doesn't matter which side is listed first; therefore, either side (A or B) can be the height, and either side can be the base.

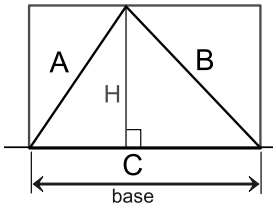
With a right triangle, the base and height are easy to see and calculate — they are the sides of the triangle that form the right angle. It is harder to understand the formula for a triangles with no 90° angle, but it still can be done.

Triangles that are not right triangles

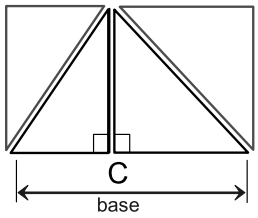
Make another triangle that has no right angles. Label the shortest side A, the middle side B, and the longest side C. Put the triangle onto a piece of typing paper with the longest side (C) as the base. The height is the distance from the top point straight down so that it forms a right angle with the base. Call the line that marks the height H. Notice that the height is not the same as any of the sides of the triangle.



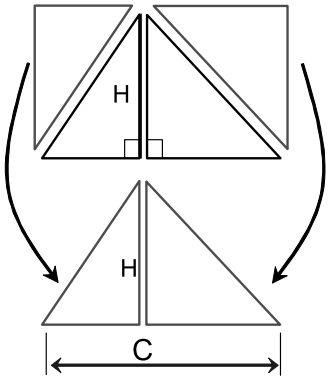
Now draw two lines parallel to H, one at each end of the triangle. Make them the same height as H. Connect the two lines at the top, so that you have drawn a rectangle that contains your original triangle. The rectangle that you just drew can be split into two rectangles, each of which contain two smaller, equal triangles.



The area of the large rectangle is $C \cdot H$. Now we need to show that this rectangle consists of two of the original triangles. *(Your child may be able to see this just by looking at the traced picture.)*



or



As shown on the right, the two triangles that were not part of the *original* triangle can be cut off and rearranged to form a triangle that is congruent to the original triangle. *(Let your child cut out the shapes and use them to form the original sized triangle.)* Therefore, because two identical triangles make up the rectangle, the area of each triangle is half of the area of the rectangle. Therefore, the area of *any* triangle is calculated by the formula:

Area = ½ base times height, or $A = \frac{1}{2} \cdot BH$

Try this with triangles of different shapes and sizes.