

Dividing by Fractions

To divide by a fraction, multiply by the reciprocal of the fraction.

$$a \div \frac{b}{c} = a \times \frac{c}{b}$$

Be sure your child has learned to multiply fractions thoroughly before you introduce division by fractions. Dividing by fractions is relatively easy to learn and teach. To divide by a fraction, you multiply by the reciprocal of the fraction.

Teaching ideas

Many children will simply accept the mathematical concept that to divide a number by a fraction, you multiply the number by the reciprocal of the fraction. Stated mathematically, this is:

$$a \div \frac{b}{c} = \frac{a}{\frac{b}{c}} = a \times \frac{c}{b}$$

Let's prove that this is true. We can multiply both sides of an equal equation by anything (like $\frac{b}{c}$) and it will stay equal.

$$\left(\frac{a}{\frac{b}{c}} \right) \times \frac{b}{c} = \left(a \times \frac{c}{b} \right) \times \frac{b}{c}$$

Rearranging, this equation can be written as follows.

$$a \times \left(\frac{\frac{b}{c}}{\frac{b}{c}} \right) = a \times \left(\frac{c \times b}{b \times c} \right) = a \times (\text{a form of } 1)$$

We end up with:

$$a \times 1 = a \times 1 \quad \text{or} \quad a = a$$

This is obviously an equal equation, so the equation we started out with is equal.

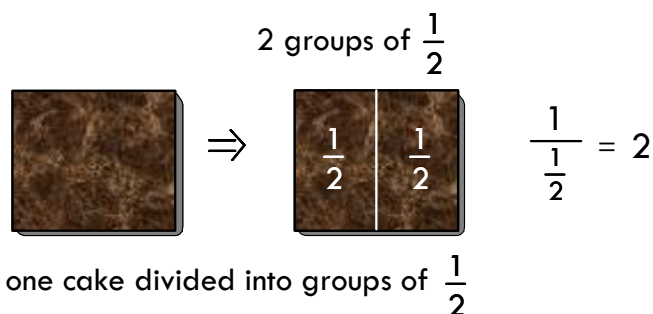
Using manipulatives to show this concept

Remember, the line in a fraction can be read “divided into groups of.” Suppose we want to do the following division problem:

$$1 \div \frac{1}{2}$$

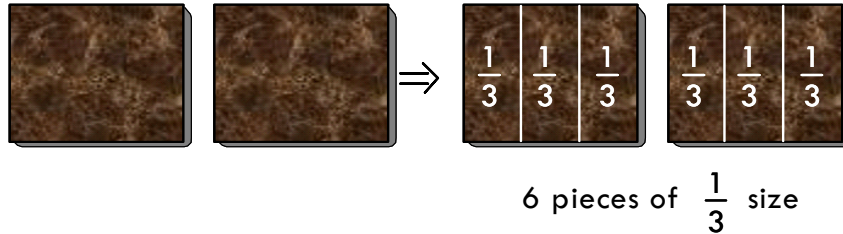
Picture a cake. This problem states that we want to take one cake and divide it into groups of one-half.

With one cake, you can make *two* groups of $\frac{1}{2}$.



If you have 2 cakes divided into groups of one-third, each cake will be divided into three pieces, so you will have six groups of one-third.

two cakes divided into groups of $\frac{1}{3}$ or $\frac{2}{\frac{1}{3}} = 6$



Practice this with your child. Draw two cakes and cut each into thirds. Point out that the denominator (3) of the fraction (1/3) shows *how many* pieces are in each cake. To get the total number of pieces from both cakes, the answer equals the denominator (3) of the dividing fraction (1/3) multiplied by the number of cakes (2).

After practicing with pictures, point out the equations:

$$\text{If } \frac{2}{\frac{1}{3}} = 6, \text{ we can also write: } 2 \times \frac{3}{1} = 6$$

(You can repeat using different numbers of cakes cut into fourths, eighths, etc.)

Multiplying a fraction by a fraction

These examples illustrate the principal that to divide by a fraction, you multiply by the reciprocal of the fraction. This works whether the numerator is a whole number or a fraction.

$\frac{3}{4} \div \frac{1}{2}$ 1 & $\frac{1}{2}$ groups of $\frac{1}{2}$

$\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1} = \frac{6}{4} = 1 \frac{1}{2}$

$$6 \div \frac{2}{3} = 6 \times \frac{3}{2} = \frac{3}{1} \times \frac{3}{2} = 9$$



6 cakes divided into groups of $\frac{2}{3}$ = 9 groups

Practice with pictures and numbers. If your child gets confused, go back to the simple examples, like one cake divided into groups of one-half.